What can a computer be commanded to do?

Sorting Complexity

Based on slides from
Lawrence Snyder
University of Washington, Seattle

Thinking About Computing

- Computers seem to run really fast … except when they don't
  - Usually we don't know why
  - Often it is communications congestion on the net
  - Other times, when saving files, say, we're waiting for the hard disk to copy everything
- Often the time a computer takes to solve a problem is proportional to how much data there is … more pixels, more time to process

Time Proportional To n

- CS folks say that problems whose work is proportional to n are n-time or linear time
  - Making an image lighter in your photo software
  - Adding a column of numbers in a spreadsheet
  - Crawling the Internet looking for links
  - … many more … linear problems are common

- Apparently some problems are not …

Searching

- Guess a number between 1 and 100. How many guesses do you need?
- Scatter a deck of cards on the floor. How many do you have to turn over to find the ace of spades?

Sorting (exchange)

- Putting a sequence of items into alphabetical or numerical order

walrus seal whale gull clam

Algorithm: compare to all following items , reorder if needed

wa se wh g c
se wa wh g c
g wa wh se c
c wa wh se g
c wa wh se g
c se wh wa g
c g wh wa se
c g wa wh se
c g se wh wa
c g se wa wh

- Other ways to sort

Sorting (bubble)

- Putting a sequence of items into alphabetical or numerical order

walrus seal whale gull clam

Algorithm: compare adjacent items, reorder if needed

Bubble sort has $\mathbf{n^2}$ Time

wa se wh g c
se wa wh g c
se wa wh g c
se wa g wh c
se wa g c wh
se wa g c wh
se wa g c wh
se g wa c wh
se g c wa wh
se g c wa wh
g se c wa wh
g c se wa wh
g c se wa wh
c g se wa wh

How Long To Sort w/Exchange?

- The pattern is, for n items
  - n-1 focus on first time
  - n-2 focus on second item
  - n-3 focus on third item
  - …
  - 1 on next to last

n-1 rows in list, avg of each row is n/2, so (n-1)×n/2

$= (n^2 - n)/2$

- Time proportional to $\mathbf{n^2}$

wa se wh g c
se wa wh g c
g wa wh se c
c wa wh se g
c wa wh se g
c se wh wa g
c g wh wa se
c g wa wh se
c g se wh wa
c g se wa wh

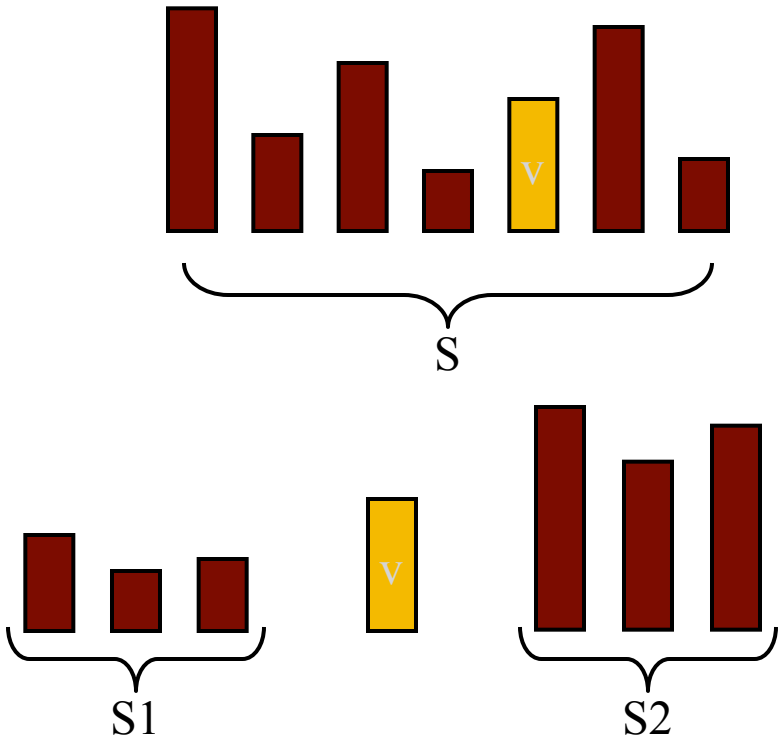
There are Different Algorithms

- Is there a better way to do sorting?

- QUICKSORT
  - Fastest known sorting algorithm in practice
  - Average case: O(N log N) (we don't prove it)
  - Worst case: $O(N^2)$
    - But, the worst case seldom happens.
  - A divide-and-conquer recursive algorithm
- Video of selection vs quicksort: http://youtu.be/cVMKXKoGu_Y

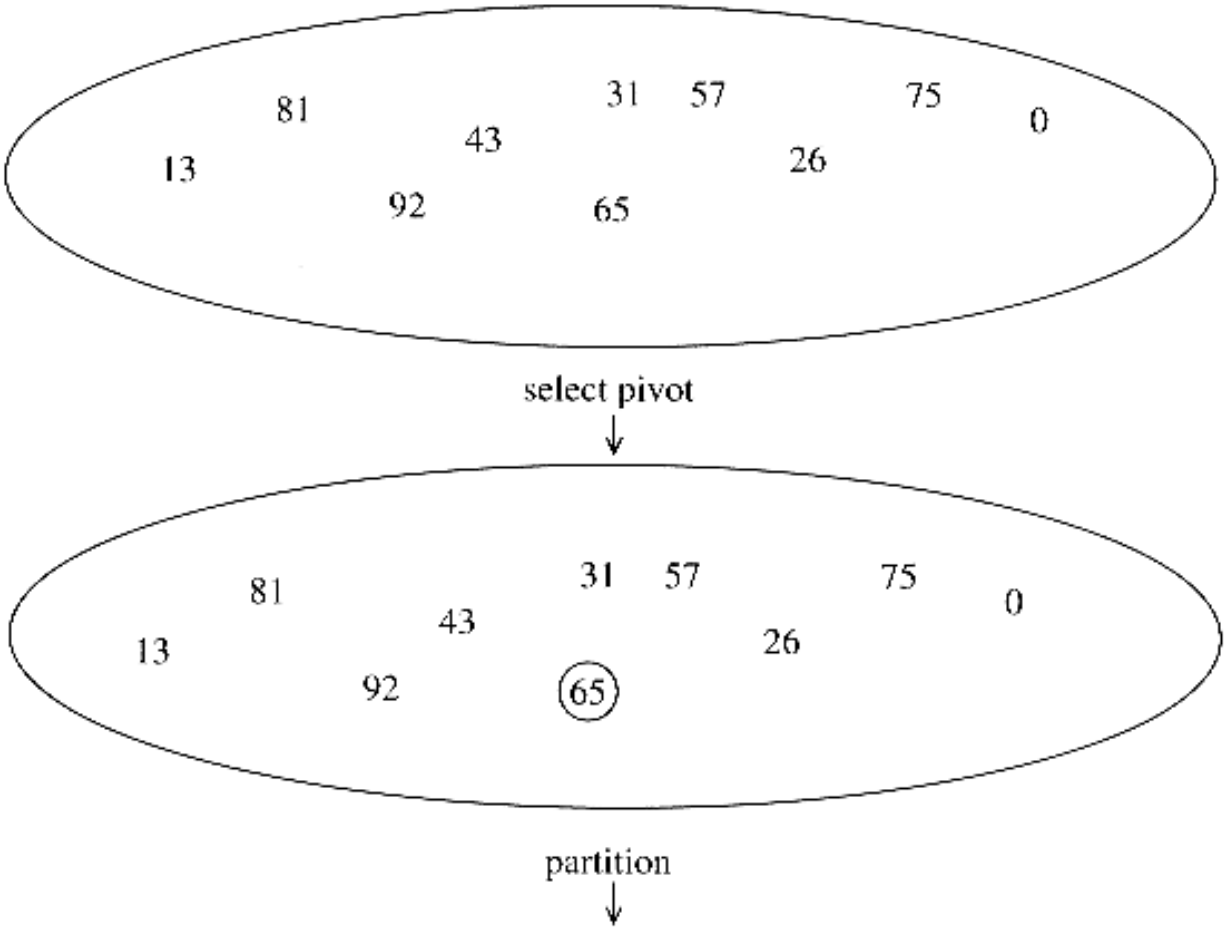
Quicksort is the best : Divide and Conquer

- Divide step:
  - Pick any element (***pivot***) v in S
  - Partition S – {v} into two disjoint groups
    S1 = {x ∈ S – {v} | x <= v}
    S2 = {x ∈ S – {v} | x ≥ v}
- Conquer step: recursively sort S1 and S2
- Combine step: the sorted S1 (by the time returned from recursion), followed by v, followed by the sorted S2 (i.e., nothing extra needs to be done)



To simplify, we may assume that we don't have repetitive elements,
So to ignore the 'equality' case!

Example


81
31
57
75
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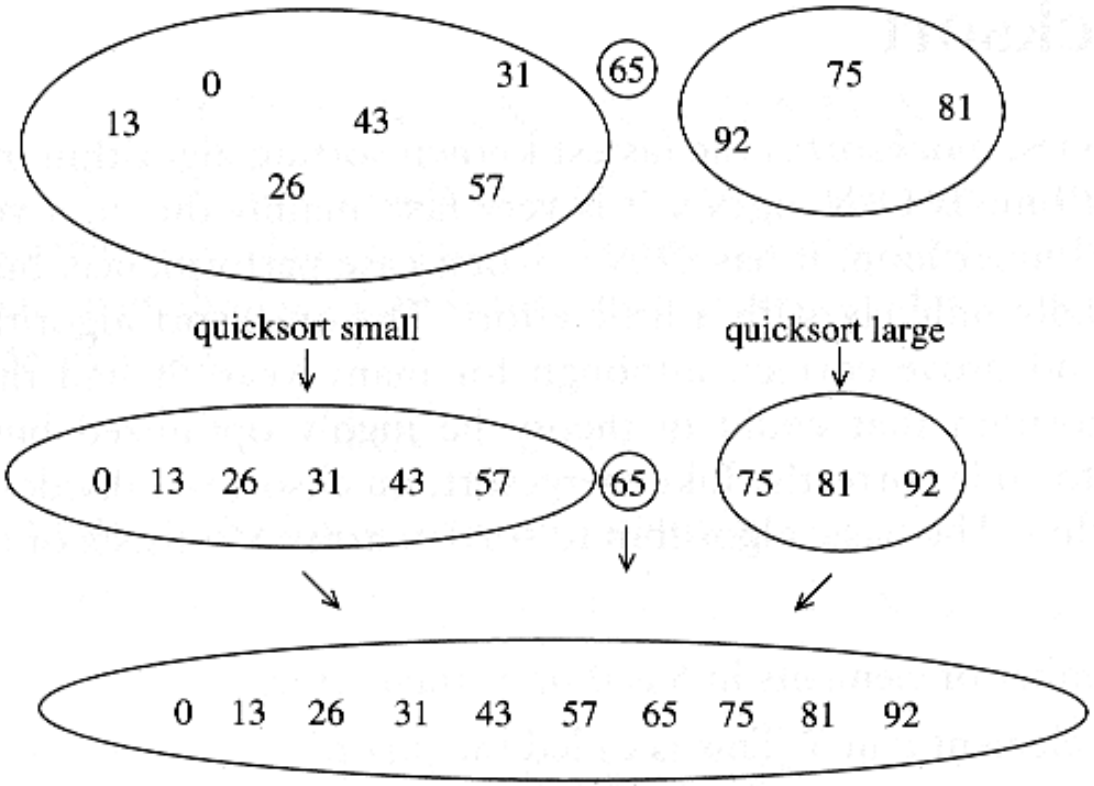
43
13
26
92
65
select pivot
81
31
57
75
0
43
13
26
92
65
partition


0
31
65
13
43
75
81
92
26
57
quicksort small
quicksort large
0 13 26 31 43 57
65
75 81 92
0 13 26 31 43 57 65 75 81 92

Polynomial

- Other computations have running time
  - proportional to n^3 – matrix multiplication
  - proportional to n^4
  - …
- All of them are lumped together as "*polynomial* time computations"
  - Considered to be realistic … a person can wait
  - Polynomial, but not linear … get a computer person to help develop your solution

Which expression grows slowest as N gets bigger?

A. Time proportional to N^2
B. Time proportional to N*log(N)

Different sorting algorithms and animations

- This one you can give different parameters for the number of things to sort, speed of animation etc.
  http://www.cs.oswego.edu/~mohammad/classes/csc241/samples/sort/Sort2-E.html
- This one has a 'dancing animation' of quick sort.
  Hungarian Folk Dance Quick Sort http://www.youtube.com/watch?v=ywWBy6J5gz8

To Infinity And Beyond

There are more complex computations …

- Suppose you want to visit 28 cities in the US (for a concert tour?) and you want to minimize how much you pay for airplane tickets
- You could select an ordering of cities (SEA → PDX → SFO → LAX …) and compute the ticket price.
- Then pick another ordering (SEA → SFO → LAX → PDX …), compute this ticket price and compare to the previous one
- Always keep the cheapest itinerary

This seems very dumb … isn't their a better way?

Traveling Salesman Problem

- Actually, no one knows a way to solve this problem significantly faster than checking all routes and picking the cheapest …
- Not polynomial time … guessing, No Poly sol'n
- This is an NP-Complete problem
  - Many many related problems … the best solution is "generate and check"
    - Best way to pack a container ship
    - Most efficient scheduling for high school students' classes
    - Least fuel to deliver UPS packages in Washington
    - Fewest public alert broadcast stations for US

Astonishing Fact

- Although there are thousands of NP-Hard Problems, meaning they're basically "generate and check" …
- NP-Complete computations (like traveling salesman) have the property that if any one of them can be done fast (n^x-time, say) then EVERYONE of the related problems can be too!

- Is Traveling Salesman solvable in n^x time is one of the great open questions in computer science

Be Famous … Answer This Question

Finding an item in an unsorted list

What is the time complexity of finding an item in an unsorted list?

A. Less than linear (log(n) or constant)
B. Linear
C. Polynomial but more time than linear
D. NP (exponential like 2^n)

There's Stuff A Computer Can't Do

- Some problems are too big – combinatorial explosion – like checking each chess game to see if there is a guaranteed win for White
  - Too many items to check
  - Doable in principle, however

Some Well Formed Problems

- One problem that has a clear specification but can't be solved is

Halting Problem

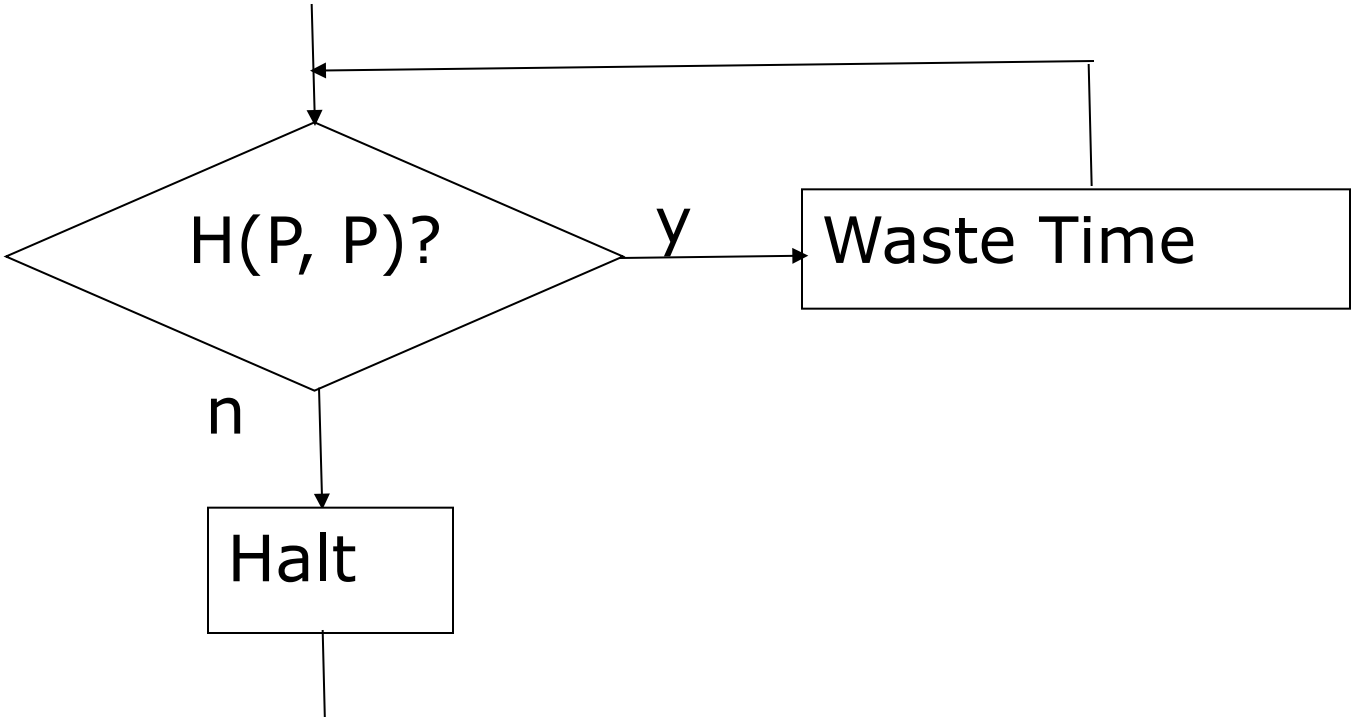
Decide, given a program P and input Q whether P(Q), that is, P run on input Q, will eventually stop running and give an answer

- This seems pretty easy … though running it won't work because it might not stop … but maybe analysis could find any errors

NOT

- The halting problem cannot be solved
- "This statement is false." True or false?
- Here's why …
  - Suppose (for purposes of contradiction) that some program H(P, Q) answers the halting problem (will it halt and give an answer) for program P on data Q
  - Notice the question is not, does it give the RIGHT answer … just will it give any answer

A New Program R(P)



An Odd Program, but what happens when we run R(R)

Two cases:

- H(R,R) says yes, it halts but by the flowchart R(R) won't halt
- H(R,R) says no, it doesn't halt but by the flowchart R(R) halts

Therefore, H cannot exist!

Summary

- We have analyzed the complexity of computations, and learned …
  - Many computations have *time proportional to n*
  - Many, like sort, have running *time proportional to* n^2
  - Others have running time proportional to n^3, n^4, …
  - Some computations are computable in principle but not in practice: *NP-complete*
  - Some things cannot be computed at all, such as the *Halting Problem*

Finding an item in a sorted list

What is the time complexity of finding an item in a sorted list?

A. Less than linear (log(n) or constant)
B. Linear
C. Polynomial but more time than linear
D. NP (exponential like 2^n)
E. Unsolvable (like the halting problem)

Summary

- The complexity of an algorithm gives an estimate of the running time increases as the size of the input to the algorithm increases.
- Good complexities are log(n), linear, or polynomial with small degree.
- Problems that have exponential times cannot be solved optimally for large inputs.
- Some problems cannot be answer (e.g. the Halting Problem).
- It is important to understand the complexity so you don't try and do the impossible.